

# What is the Higgs boson?

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School of Mathematics



Friday 22 February 2013  
(Innovative Learning Week)

Suddenly, last  
summer...

# In summary

We have observed a new boson with a mass of

**$125.3 \pm 0.6 \text{ GeV}$**

at

**$4.9 \sigma$**  significance



ATLAS today's main result (preliminary):

5.0  $\sigma$  excess at  $m_H \sim 126.5$

These accomplishments are the results of more than 20 years of talented work and extreme dedication by the ATLAS Collaboration, with the continuous support of the Funding Agencies

More in general, they are the results of the ingenuity, vision and painstaking work of our community (accelerator, instrumentation, computing, physics)

Argentina  
Armenia  
Australia  
Austria  
Azerbaijan  
Belarus  
Brazil  
Canada  
Chile  
Morocco  
Netherlands  
Norway  
Poland  
Portugal  
Romania  
Russia  
Serbia  
Slovakia

ICHEP  
Melbourne

ATLAS  
Collaboration



Japan JHR



Supporting Edinburgh's  
Festival Fringe – 35 venues  
and over 900,000 visits

162 new companies formed  
in the last five years

$$-(D_\mu \phi)^\dagger D^\mu \phi$$

$$D_\mu \phi = \partial_\mu \phi$$

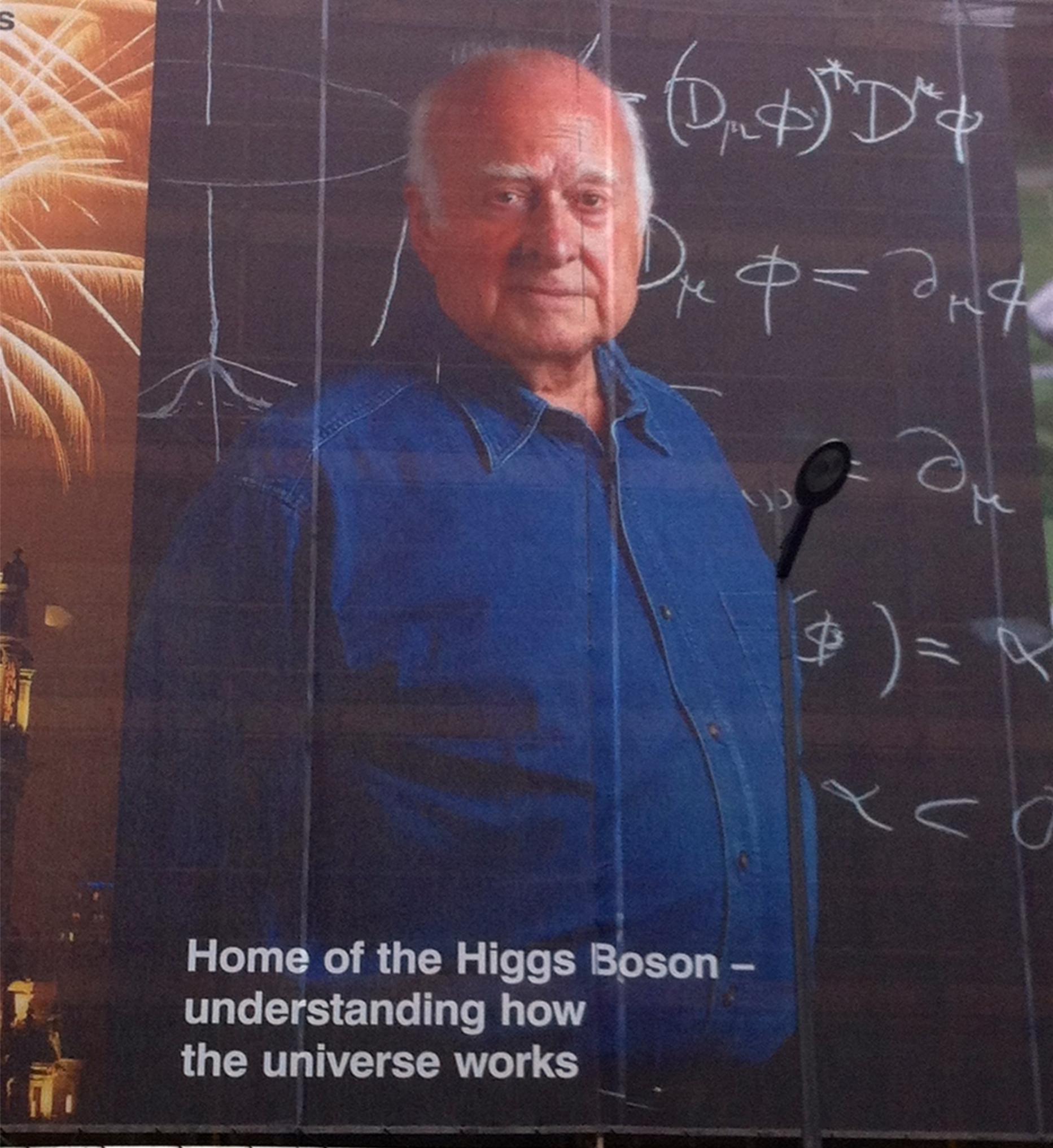
$$= \partial_\mu$$

$$(\phi) = \psi$$

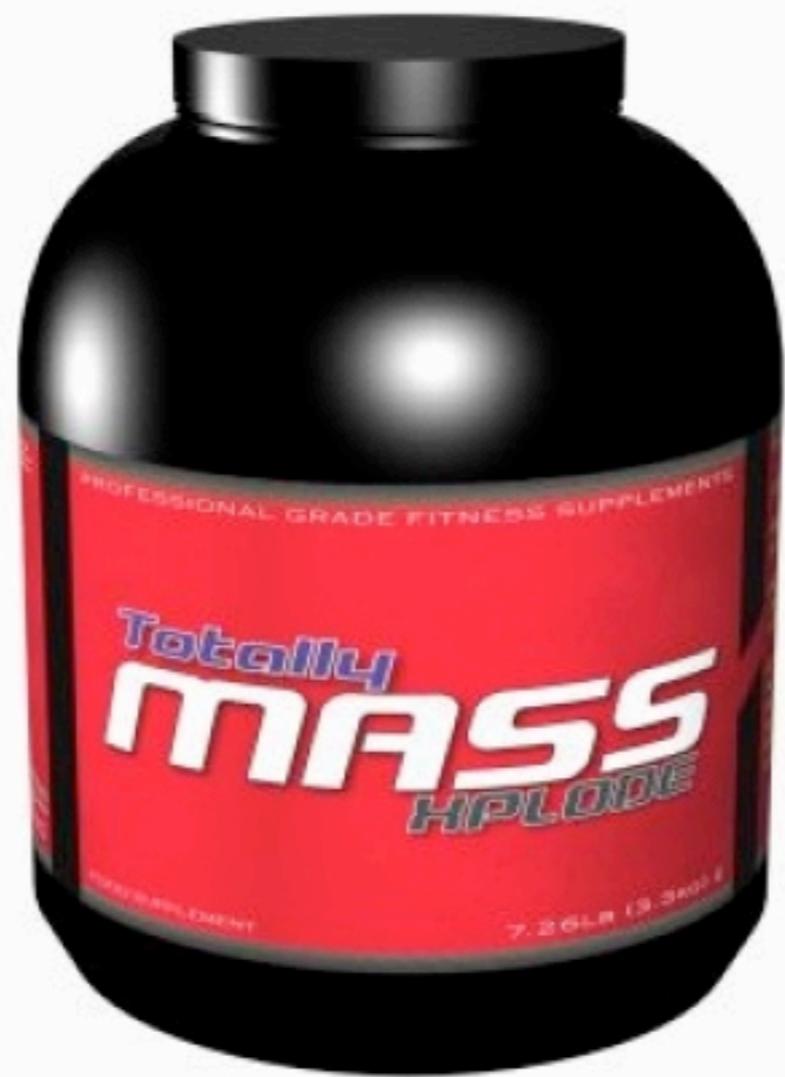
$$\psi \cup \phi$$

Home of the Higgs Boson –  
understanding how  
the universe works





**Home of the Higgs Boson –  
understanding how  
the universe works**







## BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs

Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland

(Received 31 August 1964)

In a recent note<sup>1</sup> it was shown that the Goldstone theorem,<sup>2</sup> that Lorentz-covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The purpose of the present note is to report that, as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. This phenomenon is just the relativistic analog of the plasmon phenomenon to which Anderson<sup>3</sup> has drawn attention: that the scalar zero-mass excitations of a superconducting neutral Fermi gas become longitudinal plasmon modes of finite mass when the gas is charged.

The simplest theory which exhibits this be-

about the "vacuum" solution  $\varphi_1(x) = 0$ ,  $\varphi_2(x) = \varphi_0$ :

$$\partial^\mu \left\{ \partial_\mu (\Delta\varphi_1) - e\varphi_0 A_\mu \right\} = 0, \quad (2a)$$

$$\left\{ \partial^2 - 4\varphi_0^2 V''(\varphi_0^2) \right\} (\Delta\varphi_2) = 0, \quad (2b)$$

$$\partial_\nu F^{\mu\nu} = e\varphi_0 \left\{ \partial^\mu (\Delta\varphi_1) - e\varphi_0 A_\mu \right\}. \quad (2c)$$

Equation (2b) describes waves whose quanta have (bare) mass  $2\varphi_0 \{V''(\varphi_0^2)\}^{1/2}$ ; Eqs. (2a) and (2c) may be transformed, by the introduction of new variables

$$\begin{aligned} B_\mu &= A_\mu - (e\varphi_0)^{-1} \partial_\mu (\Delta\varphi_1), \\ G_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu = F_{\mu\nu}, \end{aligned} \quad (3)$$

into the form

$$\partial_\mu B^\mu = 0, \quad \partial_\nu G^{\mu\nu} + e^2 \varphi_0^2 B^\mu = 0. \quad (4)$$

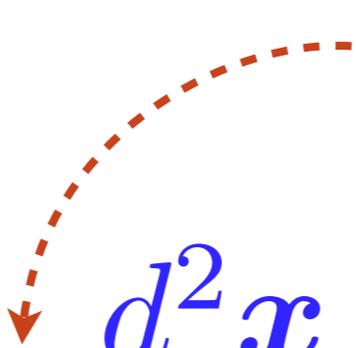
Physics: it's  
where the  
action is!



# Newton's second law

$$F = ma = m \frac{d^2 x}{dt^2}$$

inertial mass

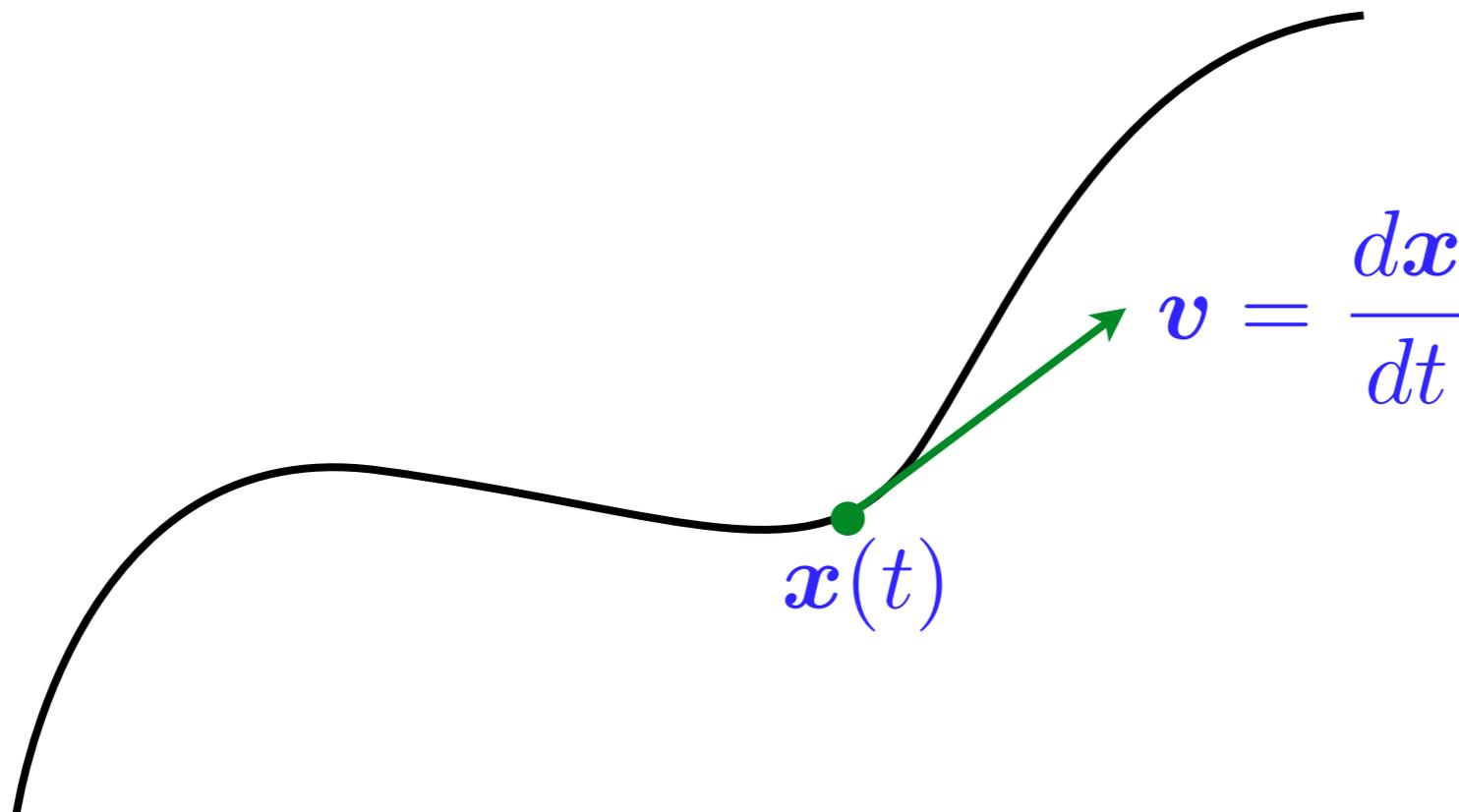


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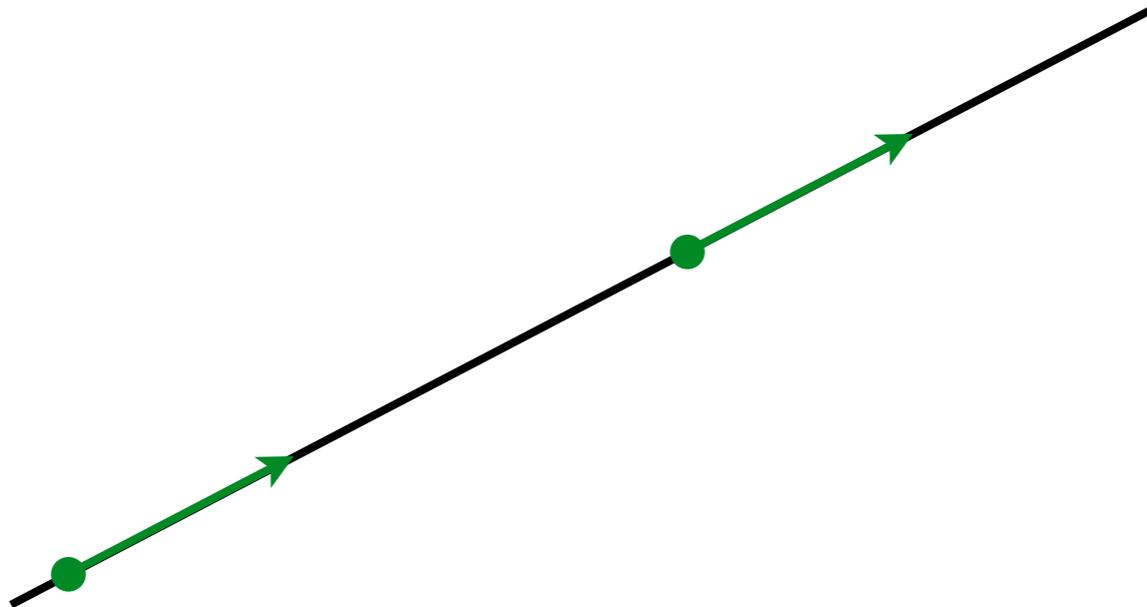
inertial mass

$x(t)$  is the path followed by the object



# Newton's first law

$F = 0 \implies$  object moves at a constant velocity





# Conservative forces

$$\mathbf{F} = -\nabla V$$

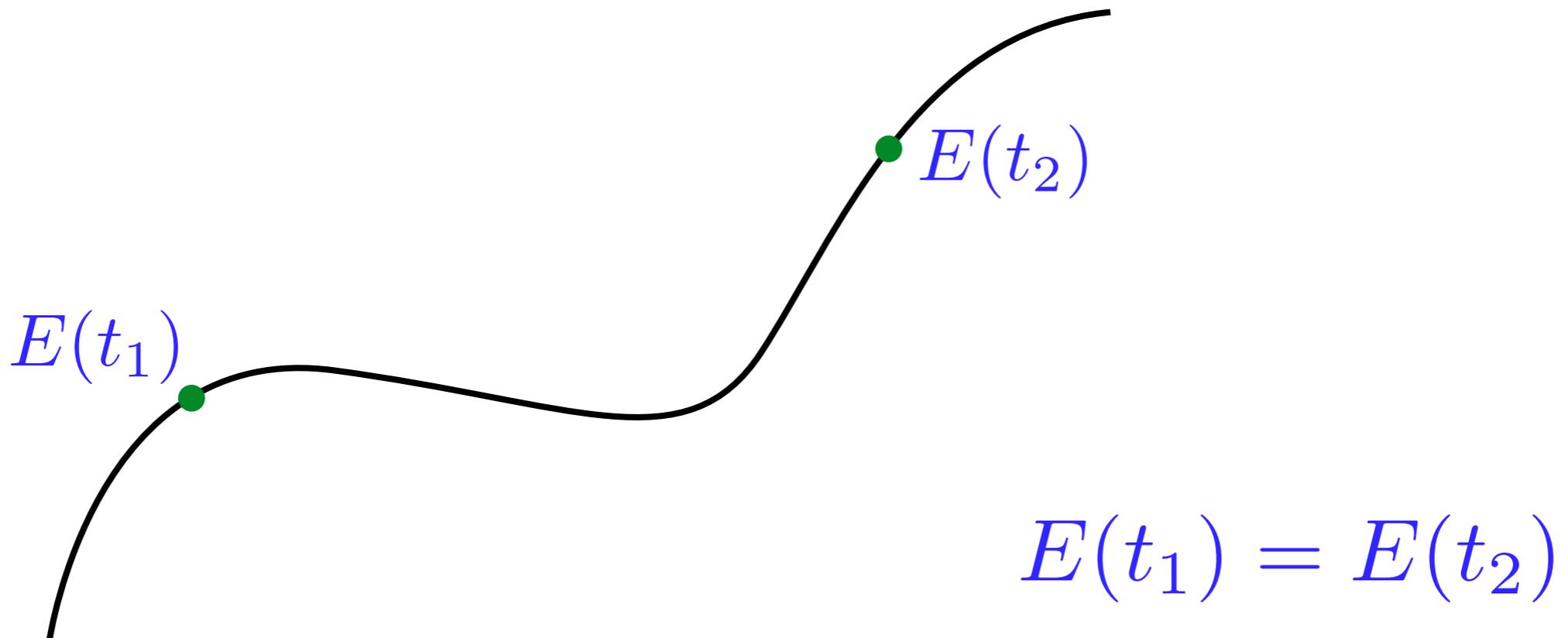
$V(\mathbf{x})$  potential function

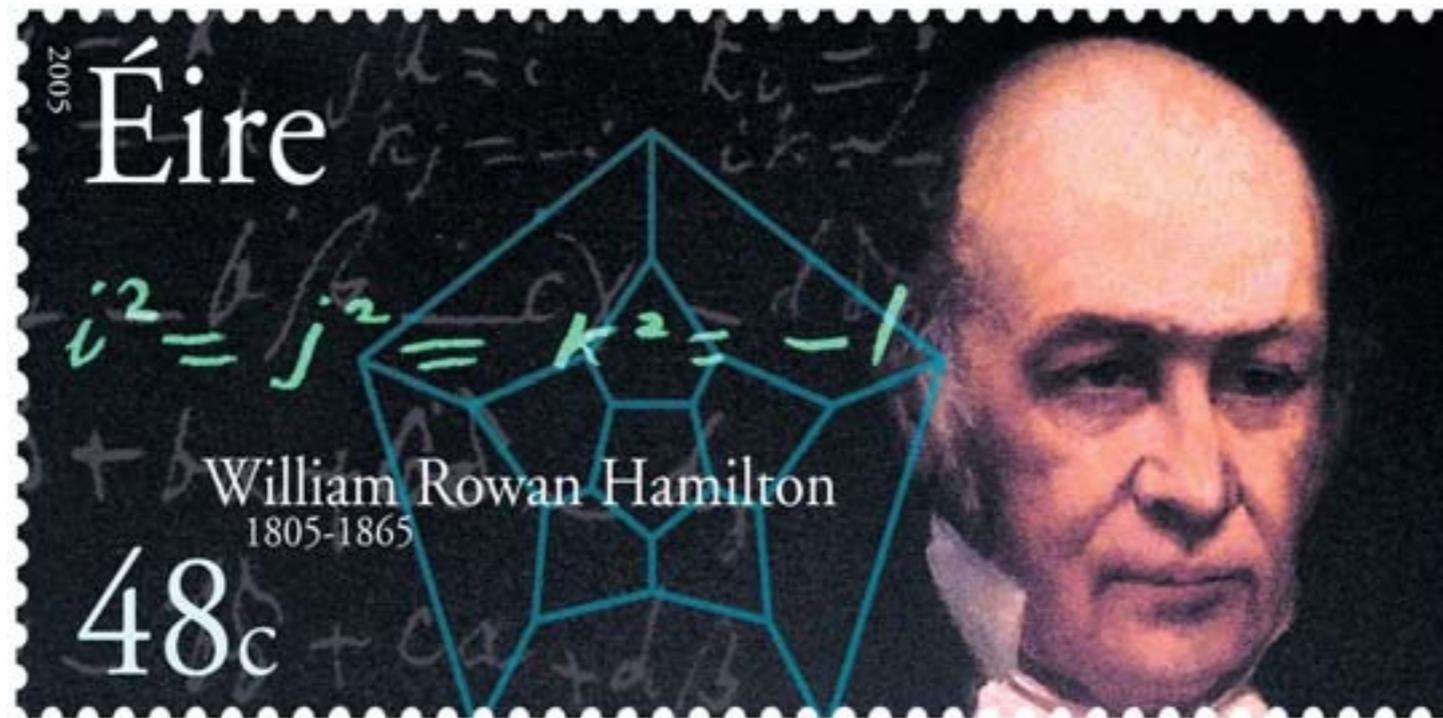
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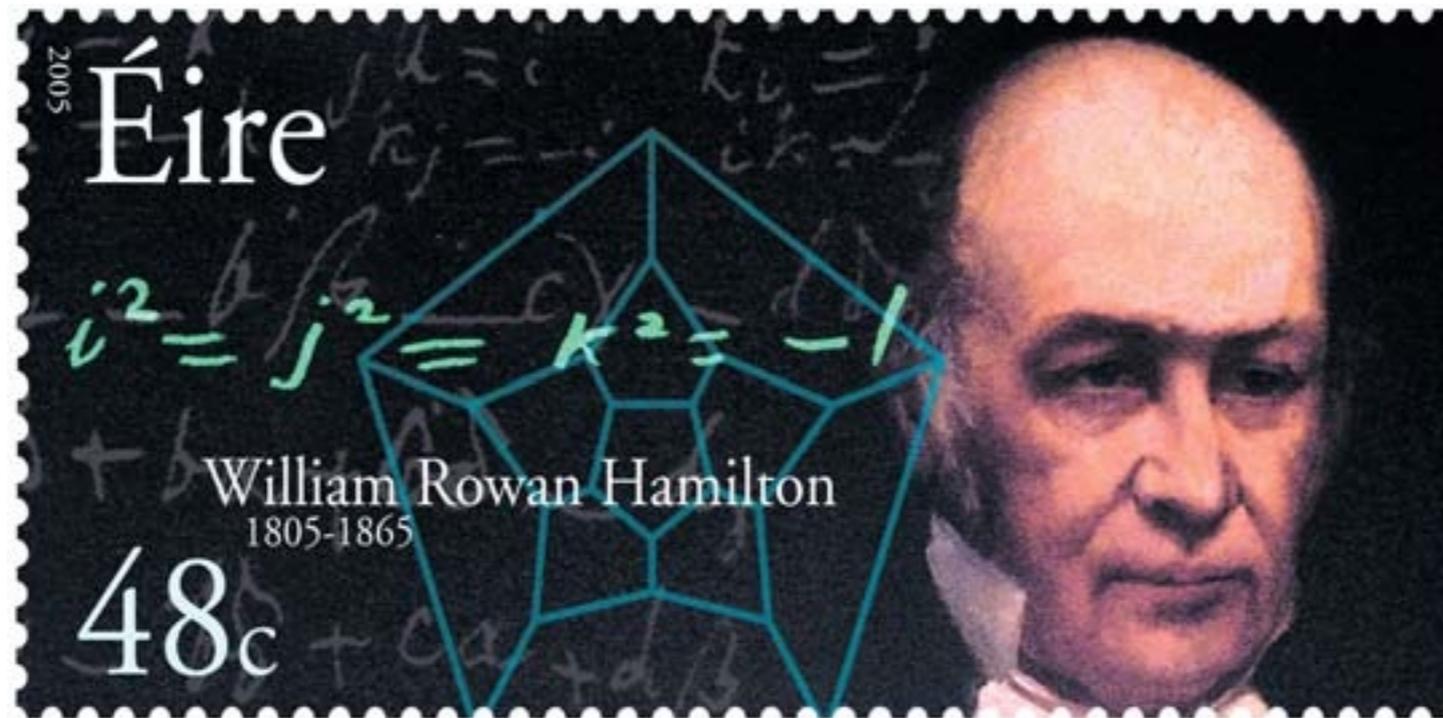
$V(\mathbf{x})$  potential function

Energy  $E = \frac{1}{2}m|\mathbf{v}|^2 + V(\mathbf{x})$  is conserved





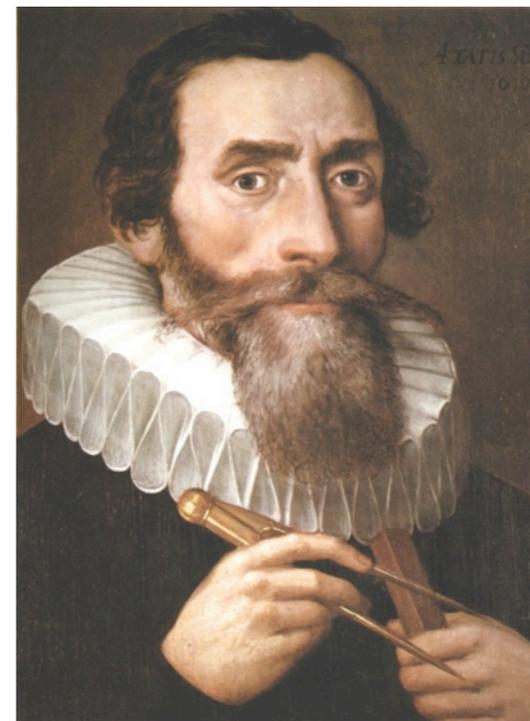
derived Newton's second law from a **principle!**



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*Nature uses as little as possible of anything.*

Johannes Kepler  
(1571-1630)

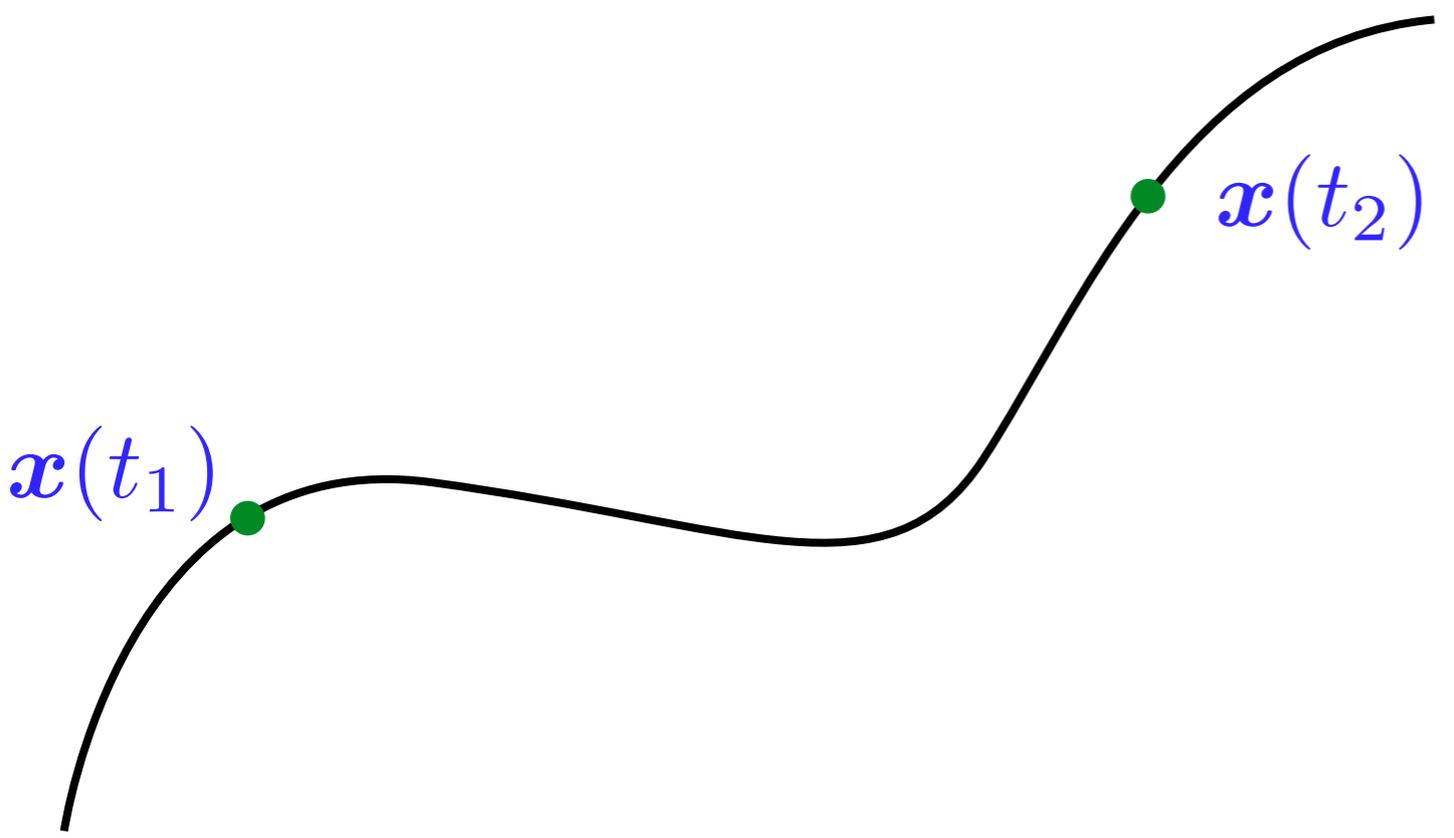


# The principle of least action

lagrangian

action

$$I = \int_{t_1}^{t_2} \left( \frac{1}{2} m |\mathbf{v}|^2 - V(\mathbf{x}) \right) dt$$

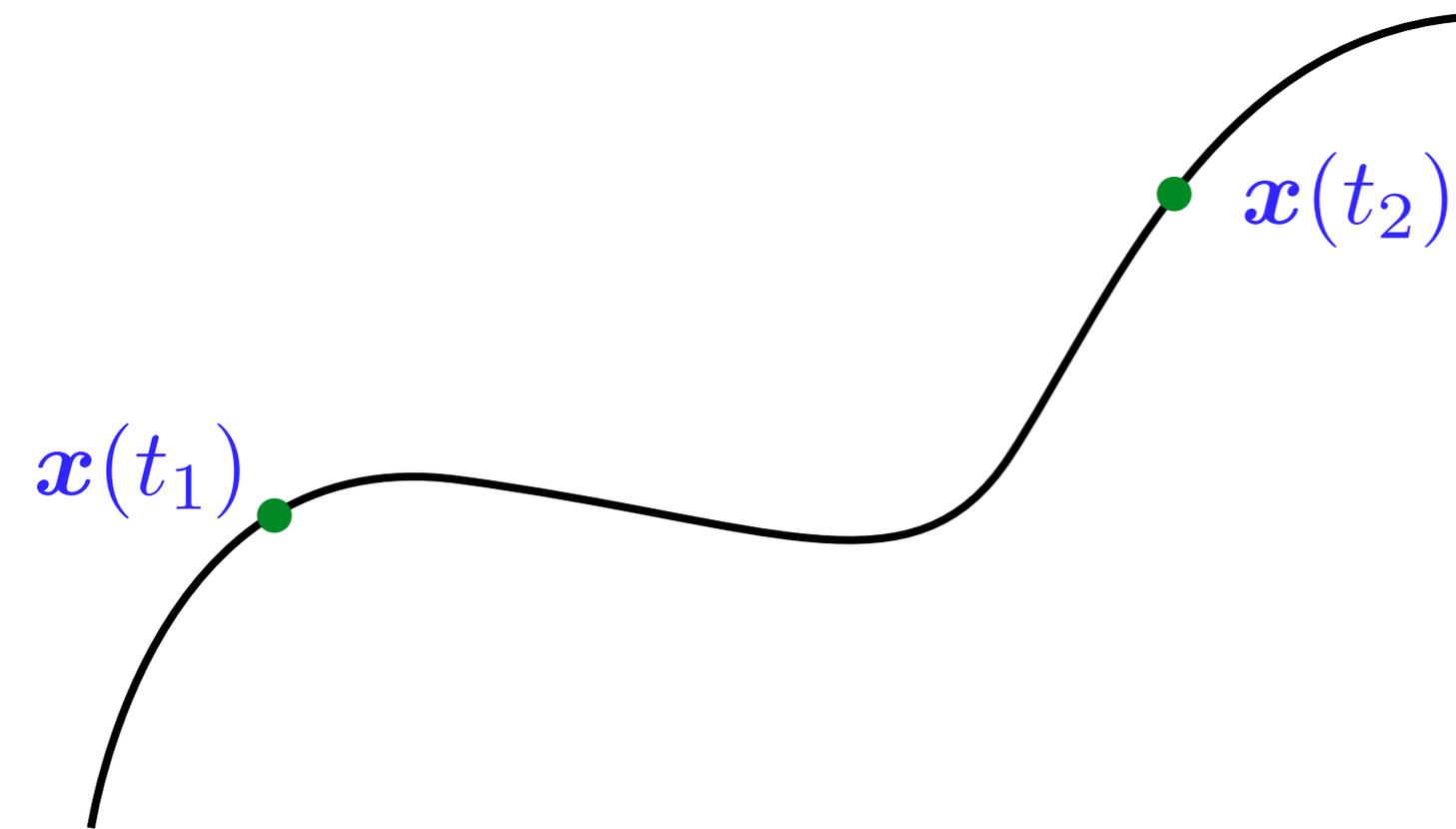


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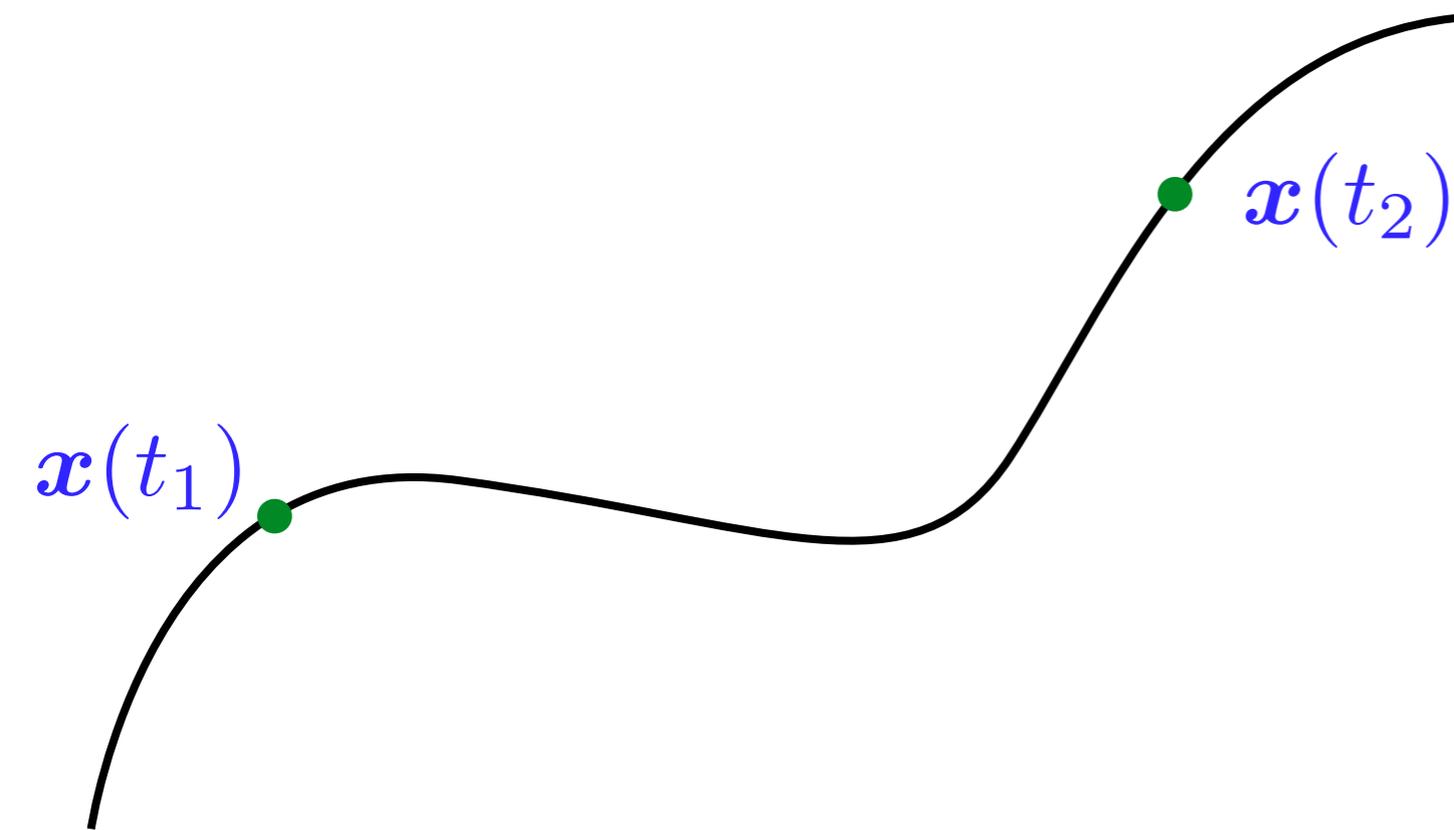
The paths with the least action satisfy Newton's second law... and conversely!

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Actions change the way we think about Physics!!

Symmetries

A transformation  $x \mapsto x'$  is called a **symmetry** if the action does not change :

$$I(x) = I(x')$$

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Symmetries take a path with least action to another path with least action (perhaps even to the same path!).

So symmetries take solutions of Newton's equation to solutions of Newton's equation.

For example,

- a **translation**  $x \mapsto x + a$

is a symmetry provided that

$$V(x + a) = V(x)$$

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- a **rotation**  $x \mapsto Rx$

is a symmetry if  $V(Rx) = V(x)$

i.e.,  $V(x) = f(|x|)$

Of course, even though the action has symmetries, solutions of Newton's equations may **break** some or all of that symmetry.

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Consider free motion:  $V(\boldsymbol{x}) = 0$

The trivial trajectory  $\boldsymbol{x}(t) = \boldsymbol{x}_0$

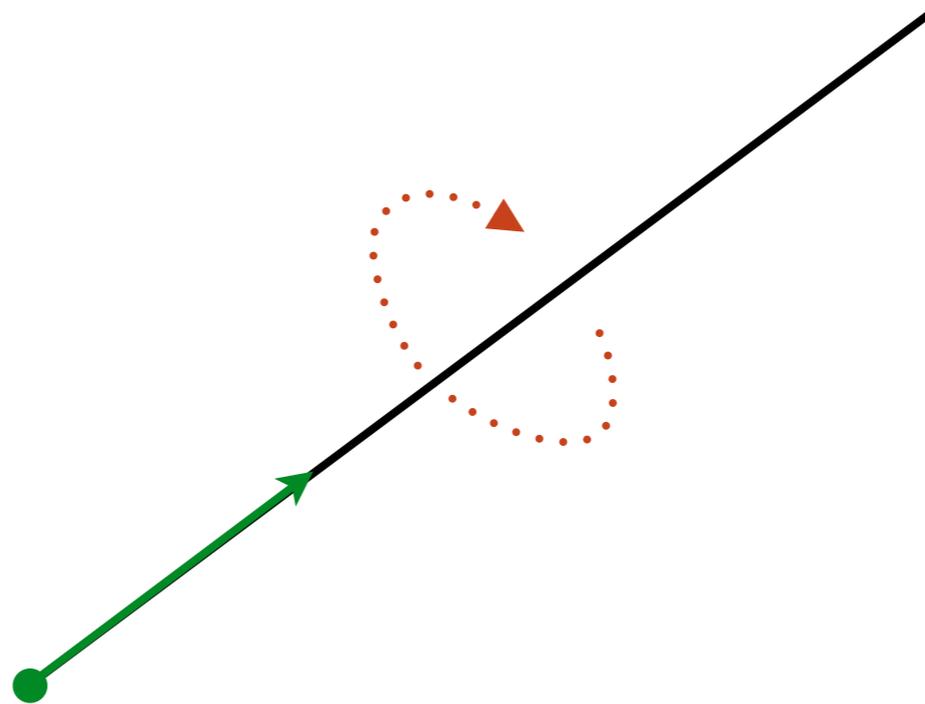
is invariant under arbitrary rotations about the point  $\boldsymbol{x}_0$

●  
 $\boldsymbol{x}_0$

If we give it a little push so that that particle is now moving at constant (nonzero) velocity

$$x(t) = x_0 + tv$$

we have broken the rotational symmetry to those with axis of rotations  $v$



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- ➔ Dynamical systems can be described by an action principle
- ➔ Solutions have least action
- ➔ Symmetries of the action take solutions to solutions
- ➔ A given solution might break some (or all) of the symmetry of the action

# Relativistic fields



And Maxwell said...

$$\begin{aligned}\nabla \cdot \mathbf{D} &= \rho \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{H} &= \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}\end{aligned}$$

and there was light!

In vacuo, Maxwell equations are

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$

where

$\mathbf{E}(\mathbf{x}, t)$  electric field

$\mathbf{B}(\mathbf{x}, t)$  magnetic field

$c$  speed of light

An immediate consequence is that the electric and magnetic fields obey the (massless) wave equation :

$$\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0 \qquad \frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0$$

(In fact, that is what light, radio waves, X-rays,... actually are.)

The wave equation can also be derived from an action.

Let us consider the scalar wave equation:

$$\frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi = 0$$

Solutions of the wave equation minimize the action:

$$I = \int dt \int d^3x \left( \frac{1}{c^2} \left( \frac{\partial \phi}{\partial t} \right)^2 - |\nabla \phi|^2 \right)$$

This action has "hidden" symmetries which mix space and time!

In fact, if we introduce a 4-vector :

$$\mu = 0, 1, 2, 3 \quad x^\mu = (x^0, \mathbf{x}) \quad x^0 = ct$$

the action becomes

$$I = \int d^4x \sum_{\mu, \nu=0,1,2,3} \eta^{\mu\nu} \frac{\partial \phi}{\partial x^\mu} \frac{\partial \phi}{\partial x^\nu} = \int d^4x \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

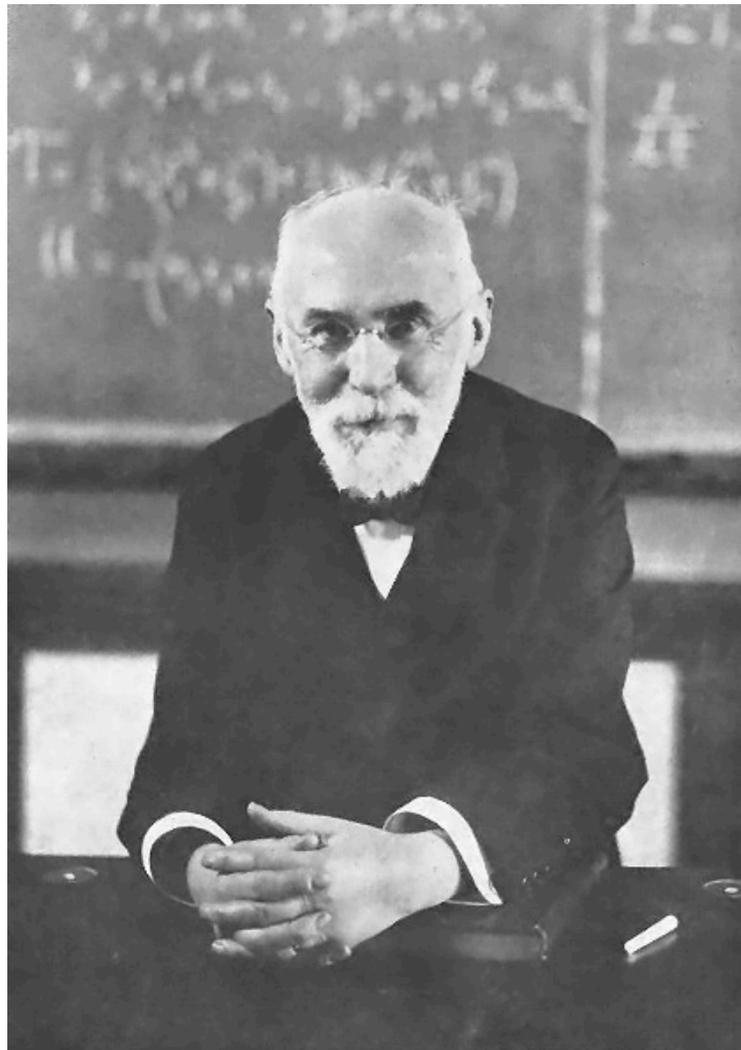
where

$$\eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \partial_\mu := \frac{\partial}{\partial x^\mu}$$

The action has relativistic symmetry:

$$\phi(x) \mapsto \phi(x') \quad (x')^\mu = \Lambda^\mu{}_\nu x^\nu \quad \Lambda^T \eta \Lambda = \eta$$

Lorentz transformations



$x^\mu$  coordinates in **Minkowski spacetime**.



"The views of space and time that I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of both will retain an independent reality."

Hermann Minkowski (1908)

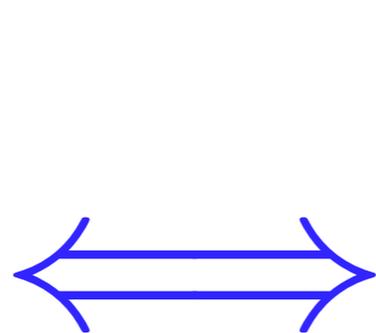
Maxwell's equations are also relativistic!

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix} \quad F_{\mu\nu} = -F_{\nu\mu}$$

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Maxwell's  
equations



$$\left\{ \begin{array}{l} \eta^{\mu\nu} \partial_\mu F_{\nu\rho} = 0 \\ \partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0 \end{array} \right.$$

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Maxwell's equations  $\iff$  
$$\begin{cases} \eta^{\mu\nu} \partial_\mu F_{\nu\rho} = 0 \\ \partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0 \end{cases}$$

They can also be derived from an action, but for an "electromagnetic potential"!

The **Bianchi identity**  $\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0$

can be solved by  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

for some  $A_\mu = (\phi, \mathbf{A})$

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Relativistic fields correspond to particles  
and  $A_\mu$  describes the **photon**

The Maxwell action (for  $A_\mu$ !)

$$I = \int d^4x \eta^{\mu\rho} \eta^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}$$

is unchanged under

→ gauge transformations

$$A_\mu(x) \mapsto A_\mu(x) + \partial_\mu \theta(x)$$

→ Lorentz transformations

$$A_\mu(x) \mapsto (\Lambda^{-1})_\mu{}^\nu A_\nu(\Lambda x)$$

Mass

For relativistic theories, the mass appears as **quadratic** terms in the action :

$$I = \int d^4x \left( \frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right)$$

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$I$  describes a massive scalar field obeying the **Klein-Gordon** equation :

$$\eta^{\mu\nu} \partial_\mu \partial_\nu \phi + m^2 \phi = 0$$

or

$$-\frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} + \nabla^2 \phi + m^2 \phi = 0$$

There is also a massive version of Maxwell equations, described by the **Proca** action:

$$I = \int d^4x \left( -\frac{1}{4} \eta^{\mu\rho} \eta^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} + \frac{1}{2} m^2 \eta^{\mu\nu} A_\mu A_\nu \right)$$

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which yields the Proca (=massive Maxwell) equation:

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The Proca action is no longer gauge invariant!

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- ➔ They are both obtained from Lorentz-invariant actions defined on Minkowski spacetime
- ➔ The Maxwell action depends on a field  $A_\mu$  which is determined only up to gauge transformations
- ➔ Mass is the quadratic term (without derivatives) in the action

# The Higgs mechanism

Actions can be combined in order to "couple" fields.

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$$\int d^4x \left( -\frac{1}{2}F^2 + \frac{1}{2}|D\Phi|^2 - V(\Phi) \right)$$

$$F^2 = \frac{1}{2}\eta^{\mu\rho}\eta^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}$$

$$|D\Phi|^2 = \eta^{\mu\nu}\overline{D_\mu\Phi}D_\nu\Phi$$

$$D_\mu\Phi = \partial_\mu\Phi + ieA_\mu\Phi$$

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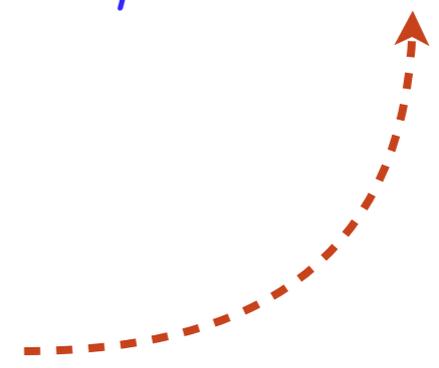
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electric charge



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And this is also a symmetry of the third term, provided that

$$V(\Phi) = f(|\Phi|)$$

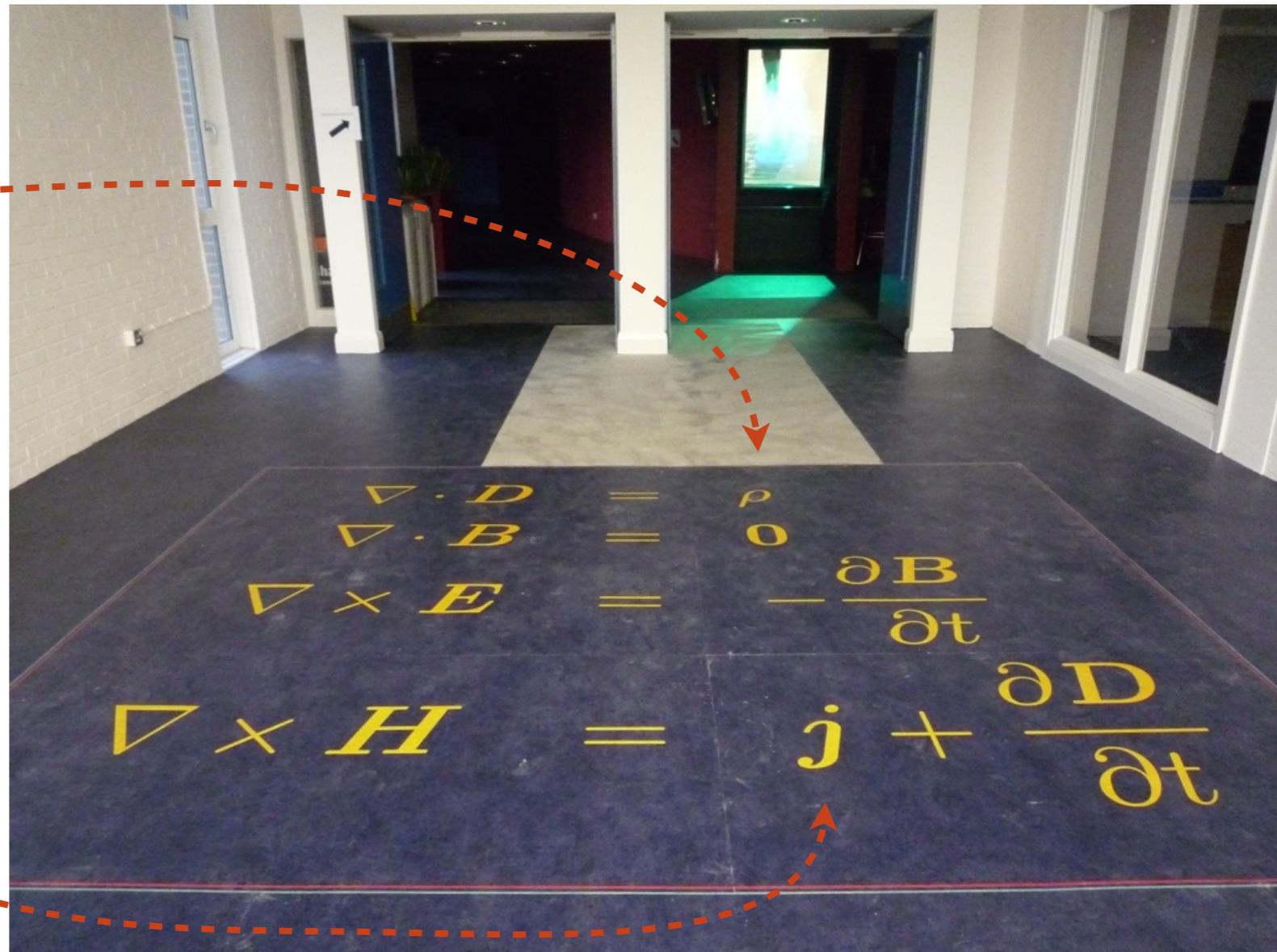
The Maxwell equation is modified by an "electric" current term :

$$\eta^{\lambda\mu} \partial_\lambda F_{\mu\nu} = J_\nu = \frac{1}{2} e (\bar{\Phi} \partial_\nu \Phi - \Phi \partial_\nu \bar{\Phi}) - e^2 A_\nu |\Phi|^2$$

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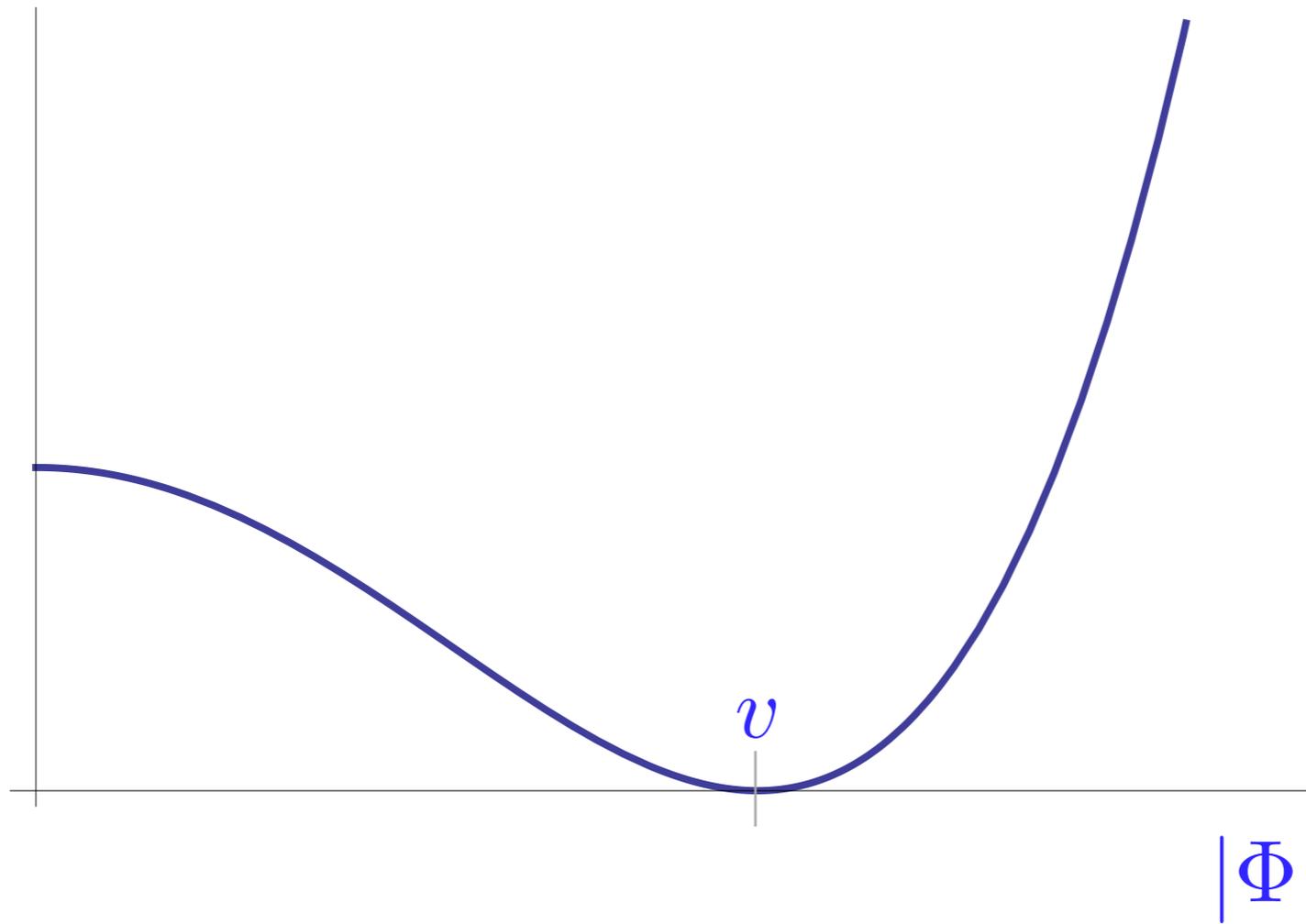
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$$J_\nu = (\rho, \mathbf{j})$$



# Potential

$$V(\Phi) = f(|\Phi|)$$

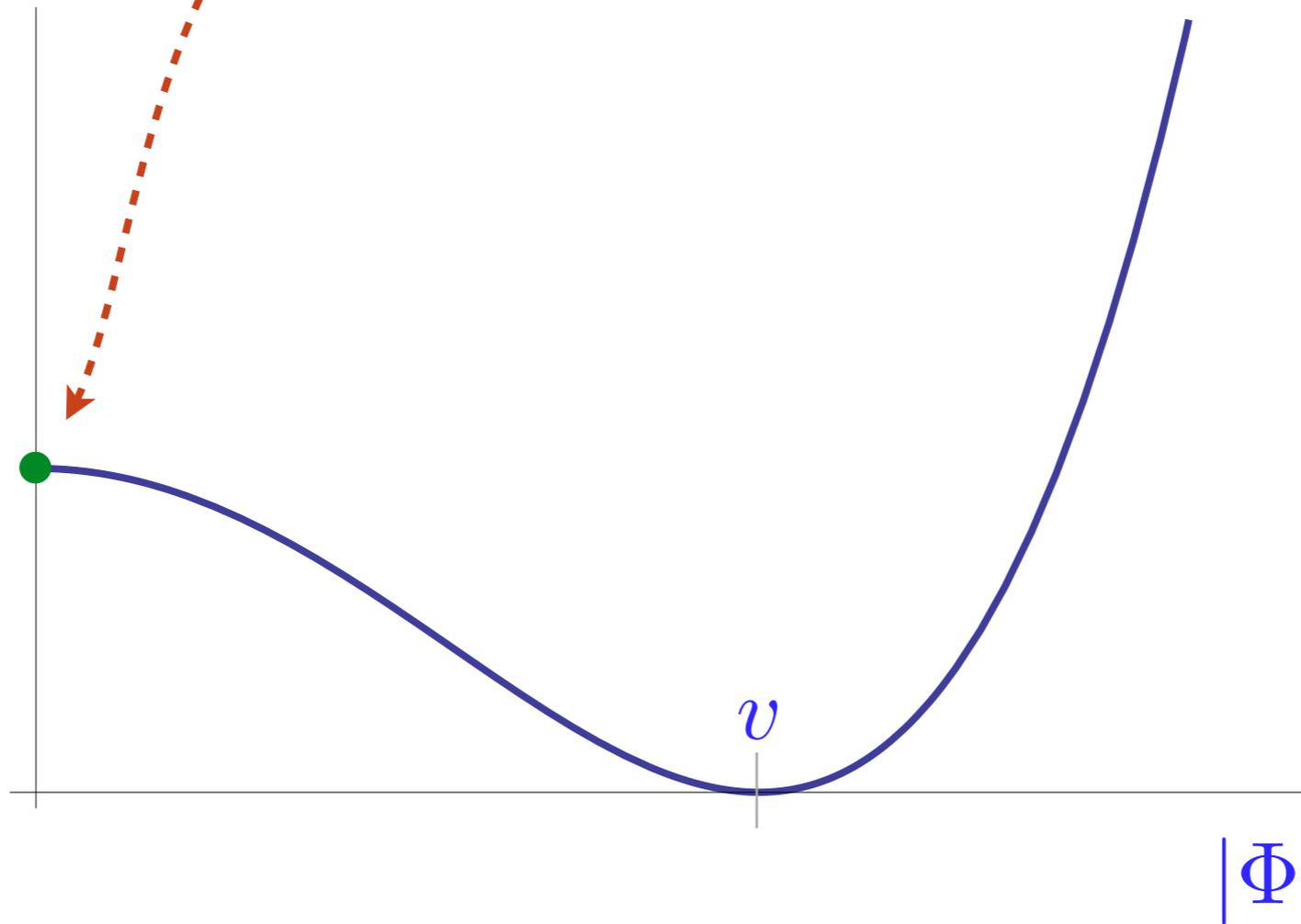


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unstable critical point

$$|\Phi| = 0$$

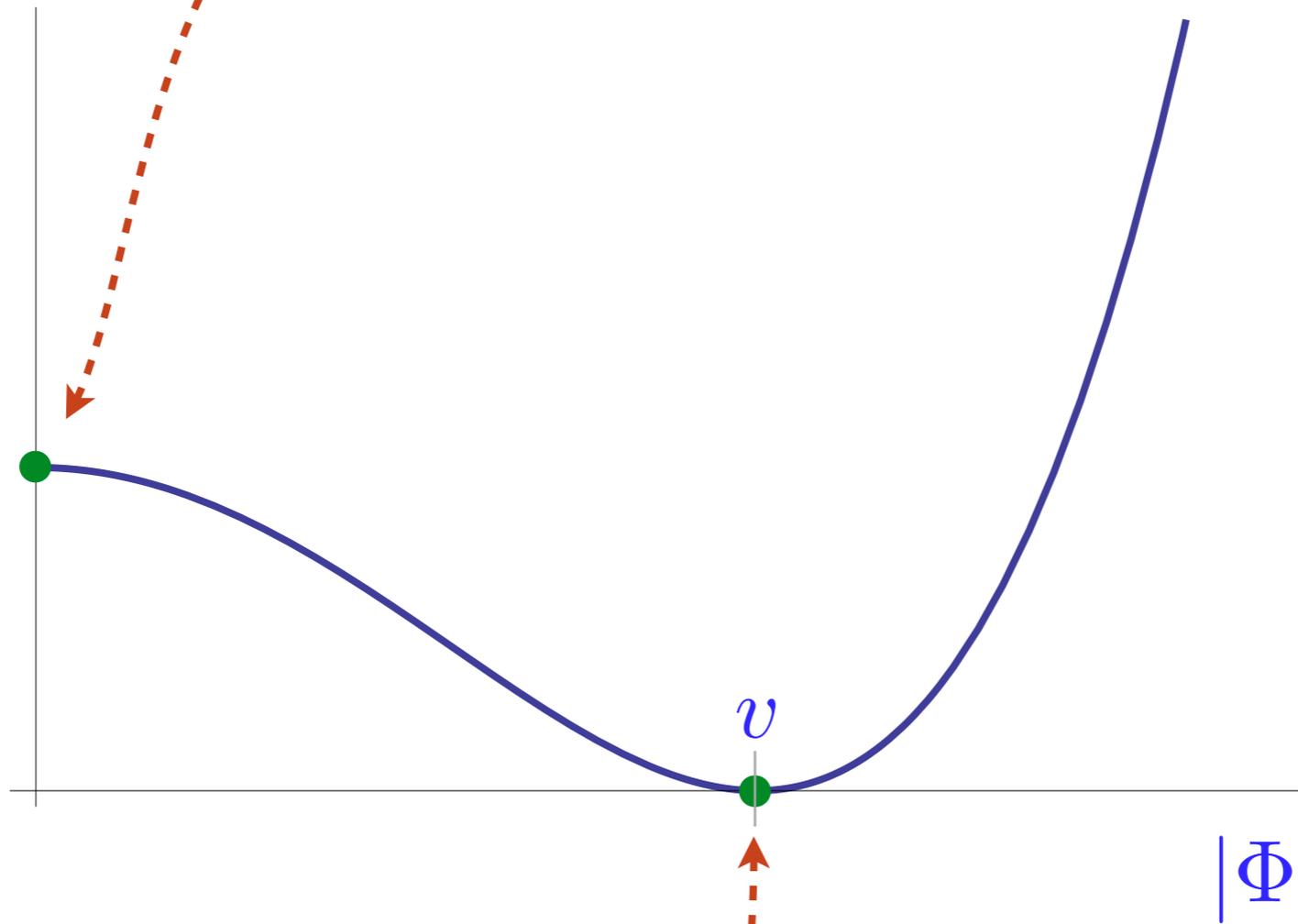


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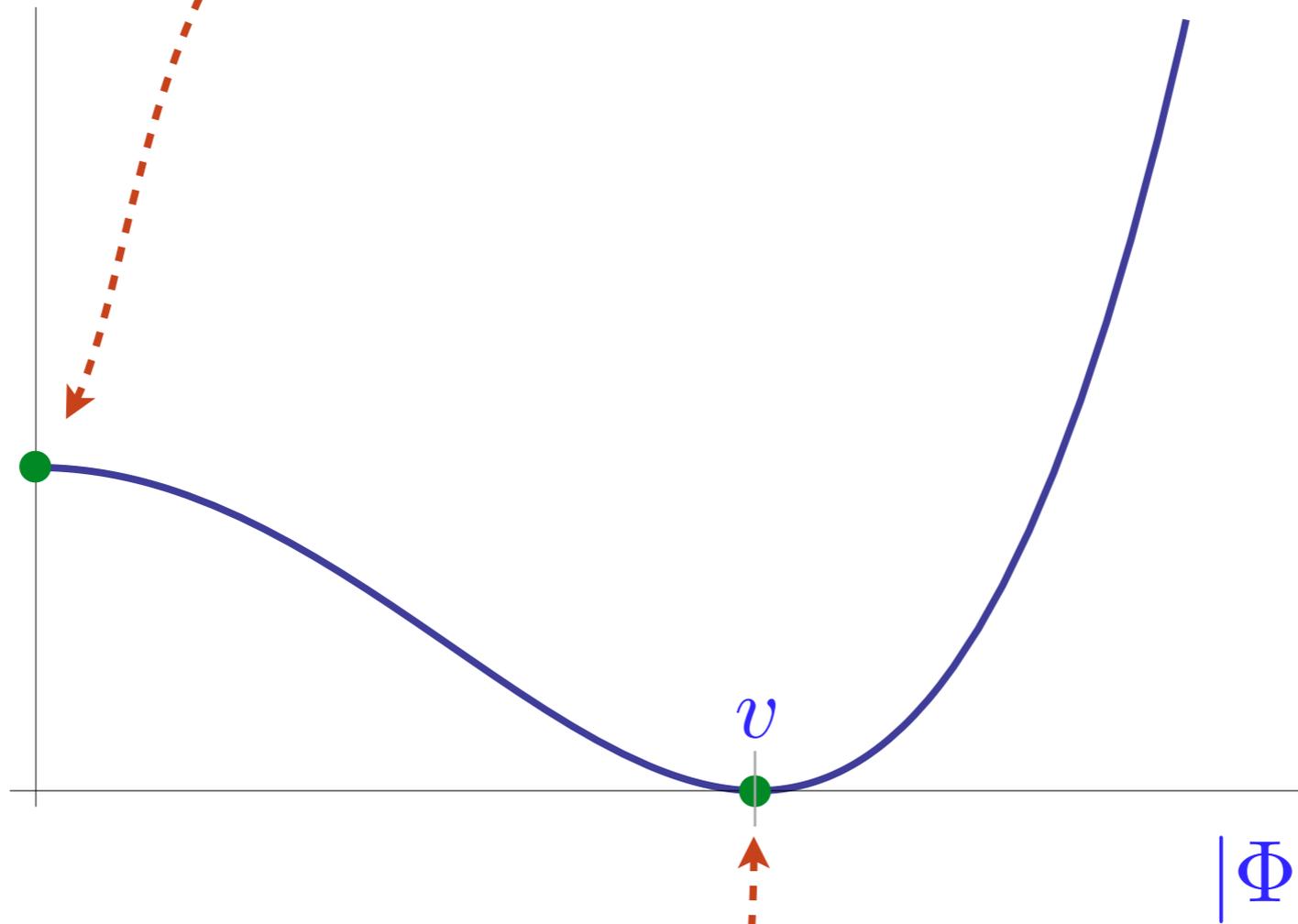
$$|\Phi| = v$$

stable critical points

# Potential

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unstable critical point  
symmetric  $|\Phi| = 0$

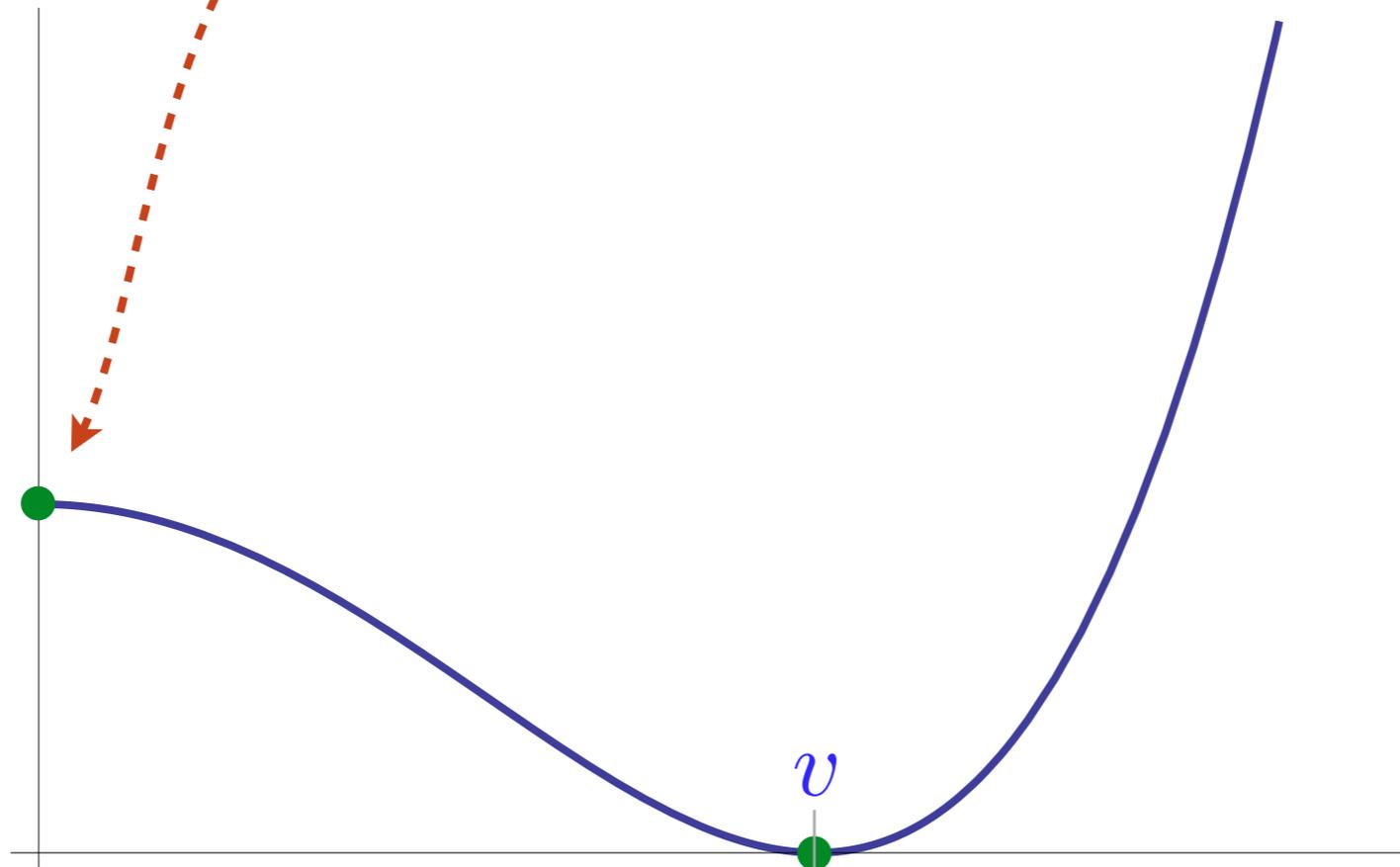


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stable critical points

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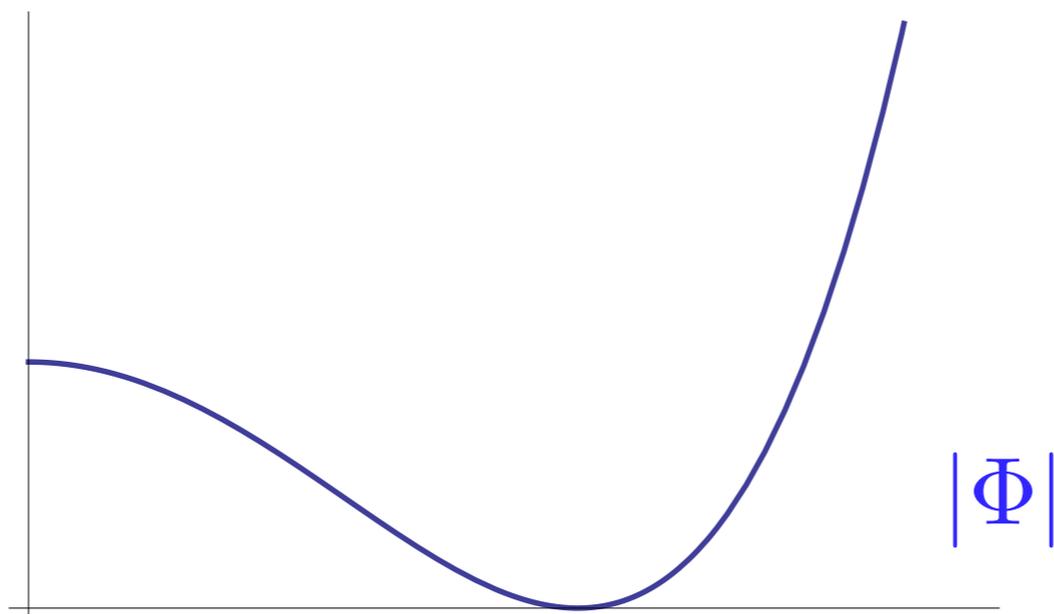
unstable critical point  
symmetric  $|\Phi| = 0$

$$|\Phi| = v$$

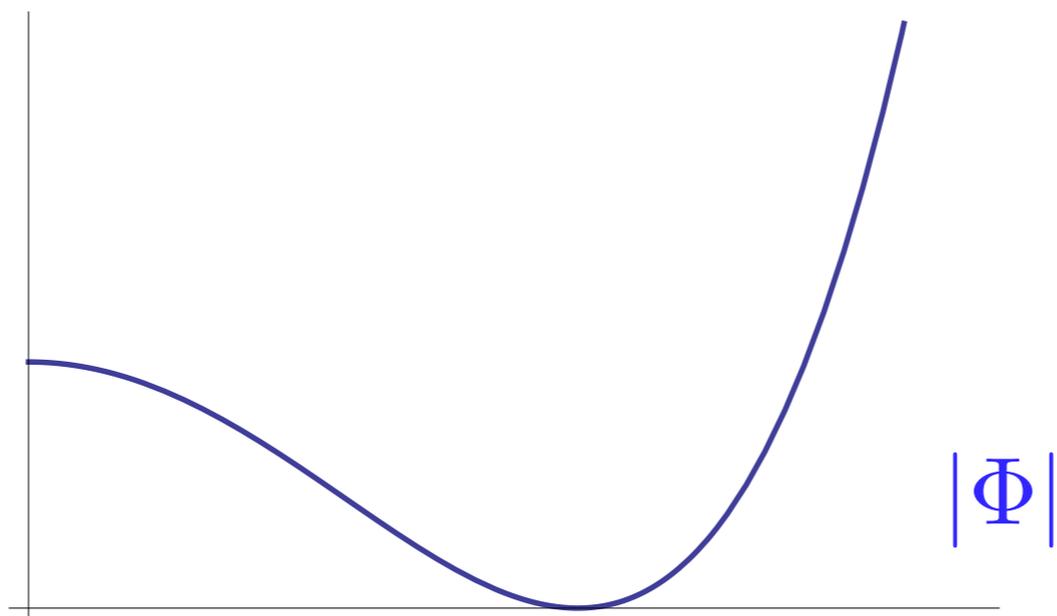
stable critical points

symmetry is broken!

$$V(\Phi) = f(|\Phi|)$$

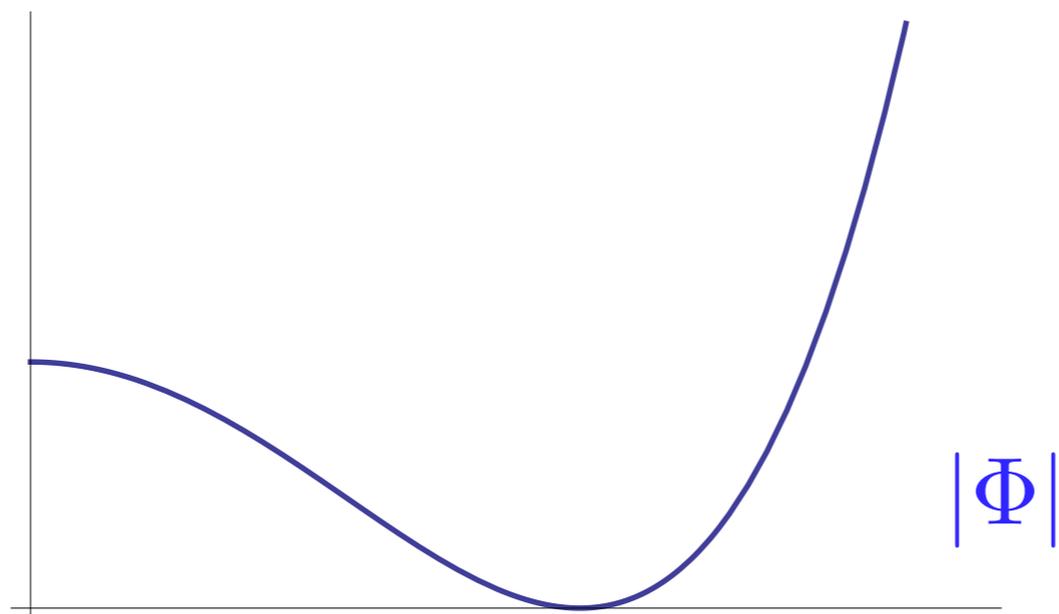


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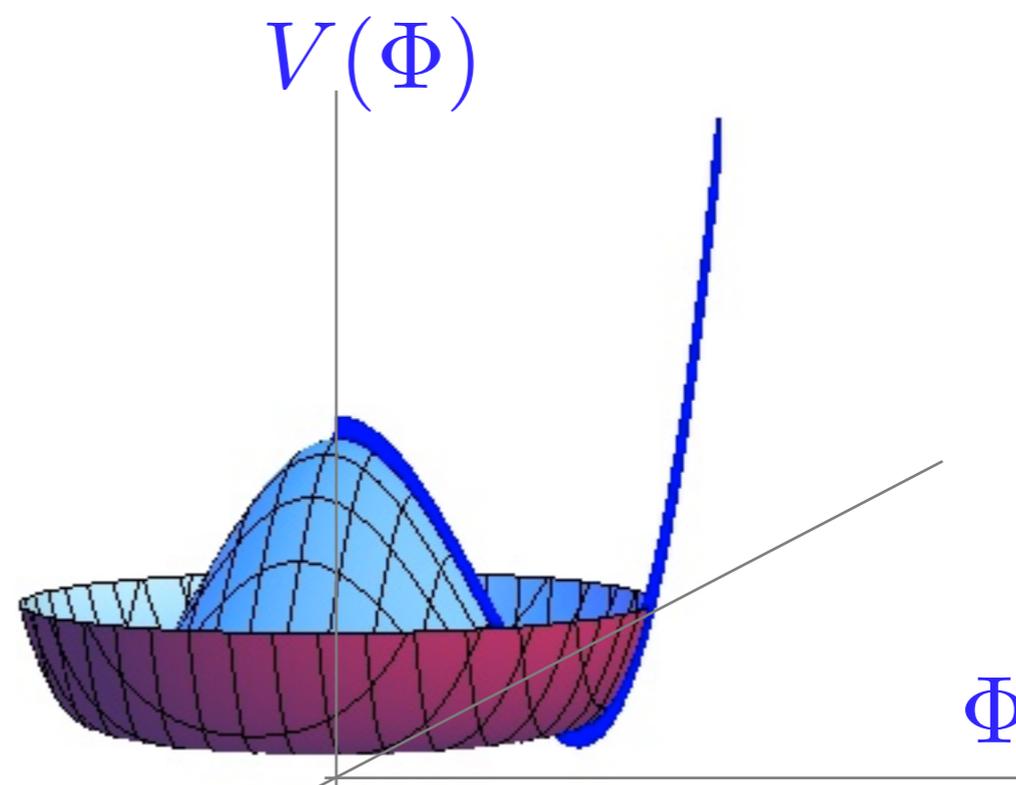


$$\Phi = |\Phi|e^{i\theta}$$

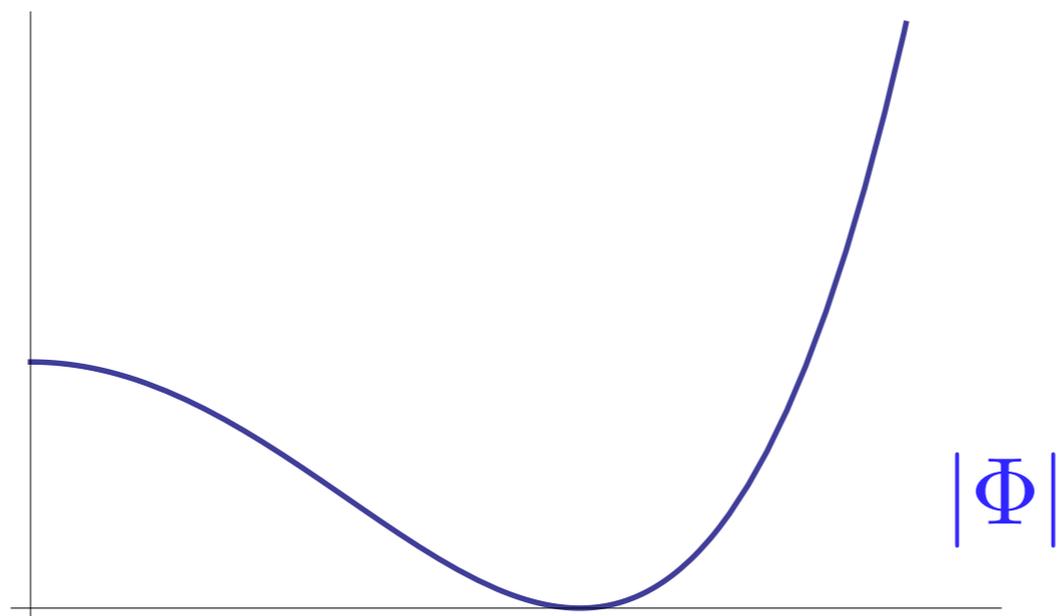
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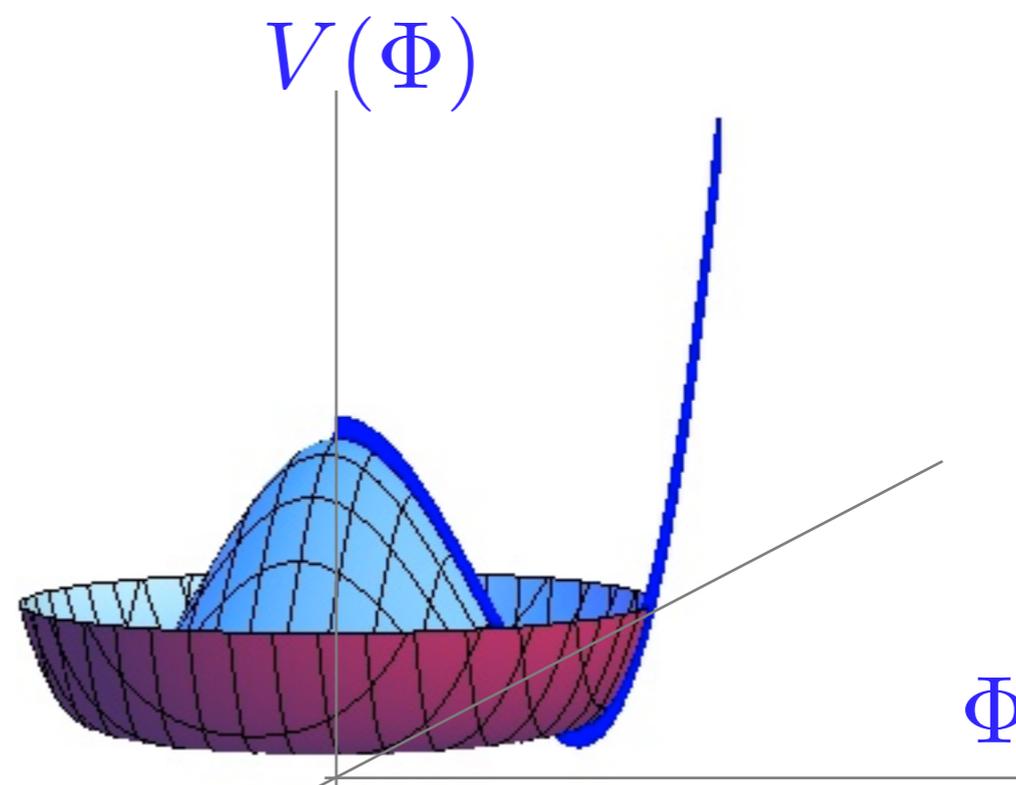
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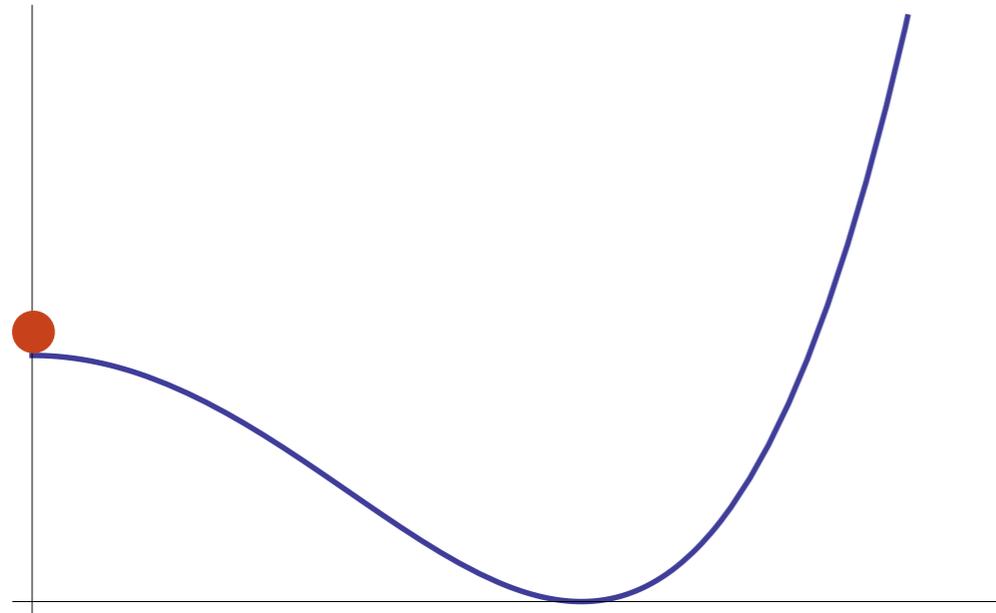


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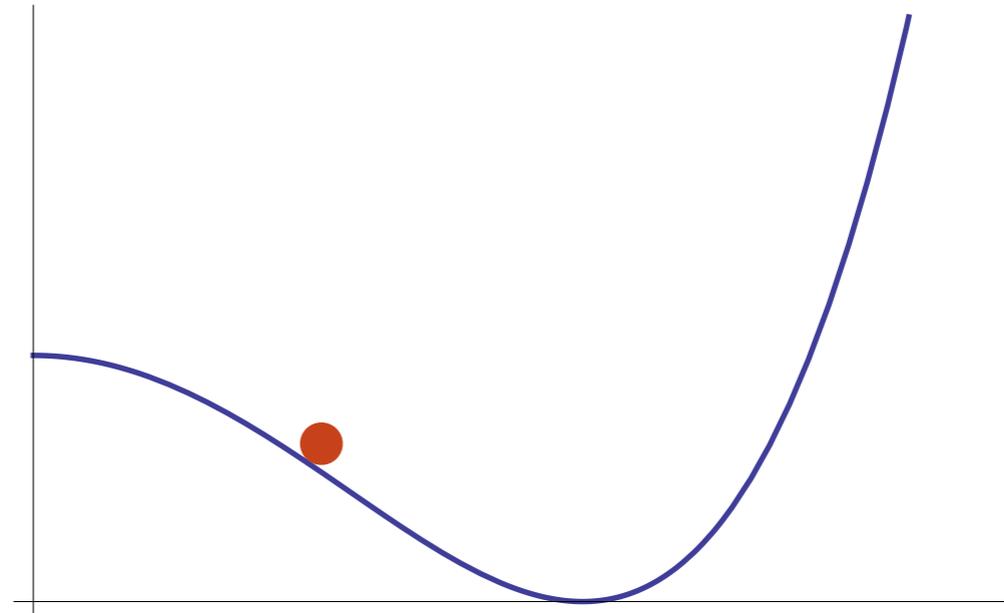
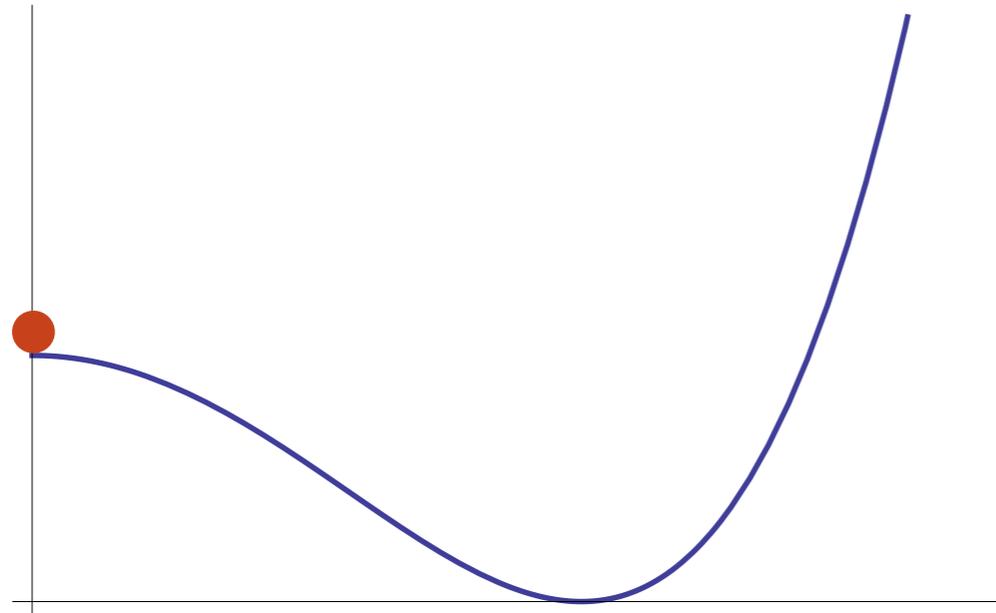


Imagine a ball rolling on this potential:

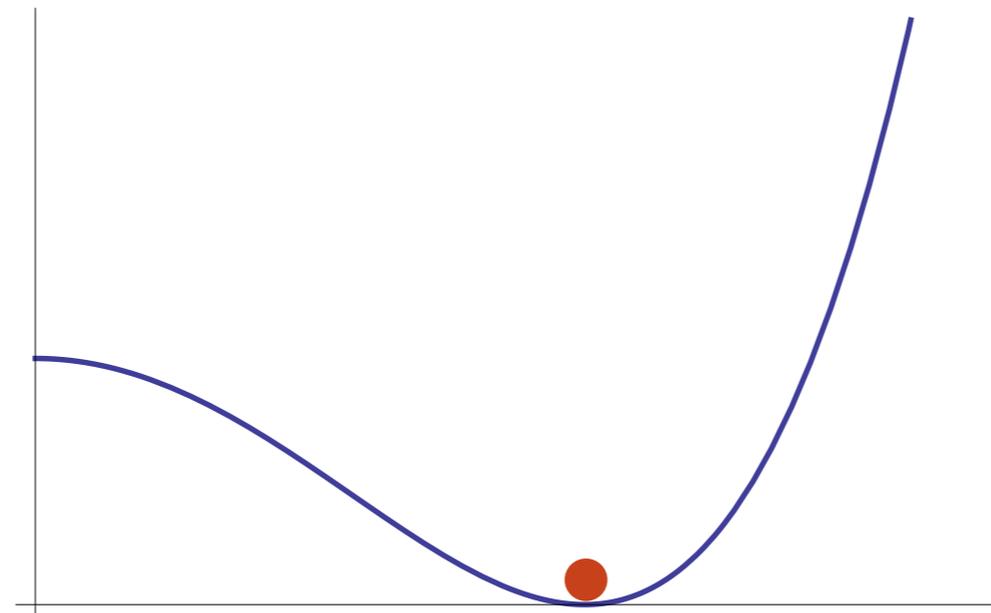
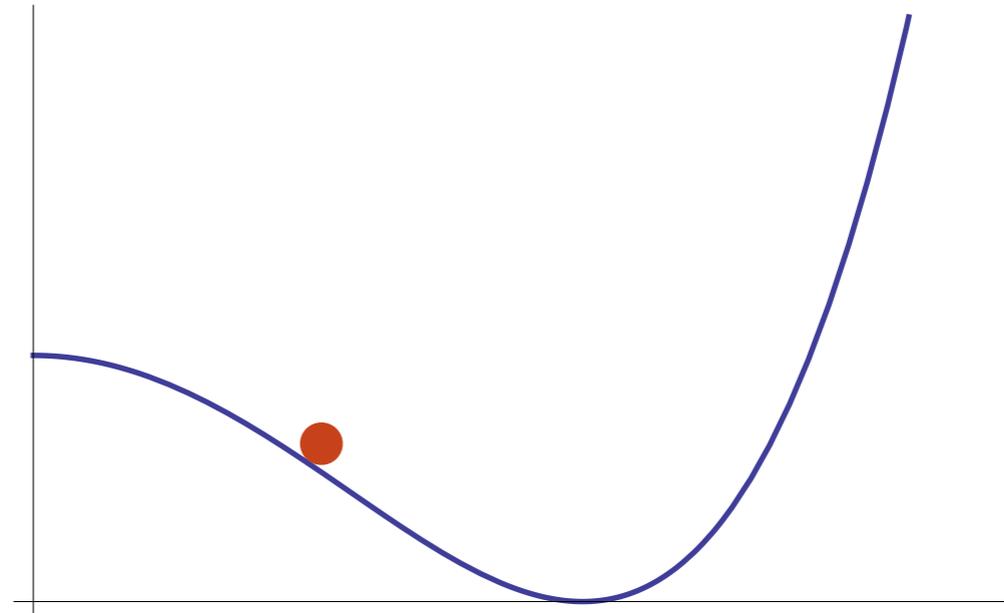
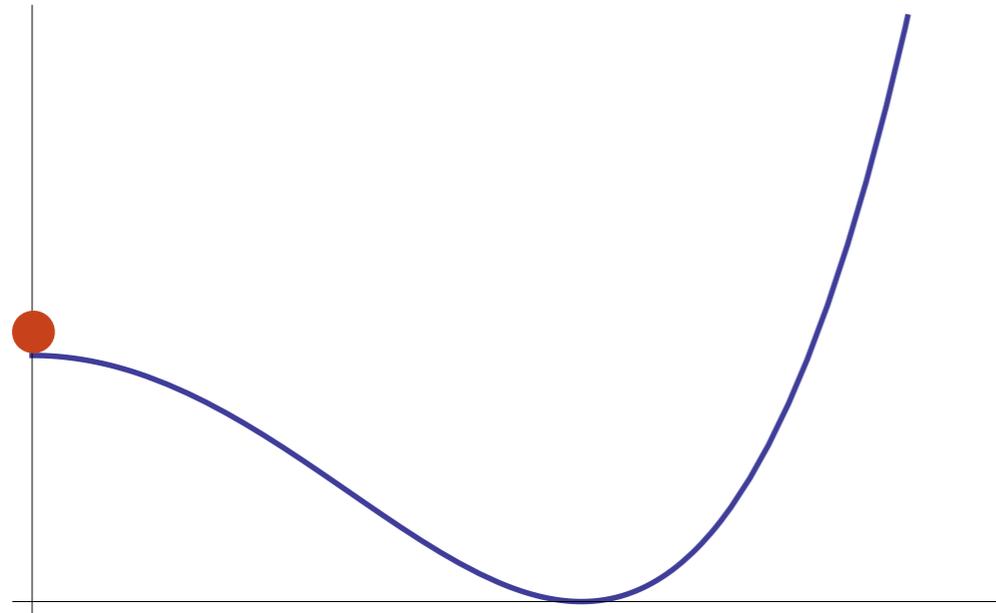
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Expanding about the unstable critical point, we see a massless photon and a "tachyonic" scalar.



imaginary mass

$$m^2 = f''(0) < 0$$

Nature dislikes unstable critical points and quantum/thermal fluctuations will move the system away from them and towards a stable critical point.

Let's expand about a stable critical point with  $|\Phi| = v$

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we obtain...

$$I = \int d^4x \left( -\frac{1}{2}F^2 + \frac{1}{2}e^2v^2\eta^{\mu\nu}B_\mu B_\nu \right. \\ \left. + \frac{1}{2}\eta^{\mu\nu}\partial_\mu h\partial_\nu h - \frac{1}{2}V''(v)h^2 + \dots \right)$$

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The photon "ate" one of the two scalars  
and became massive!

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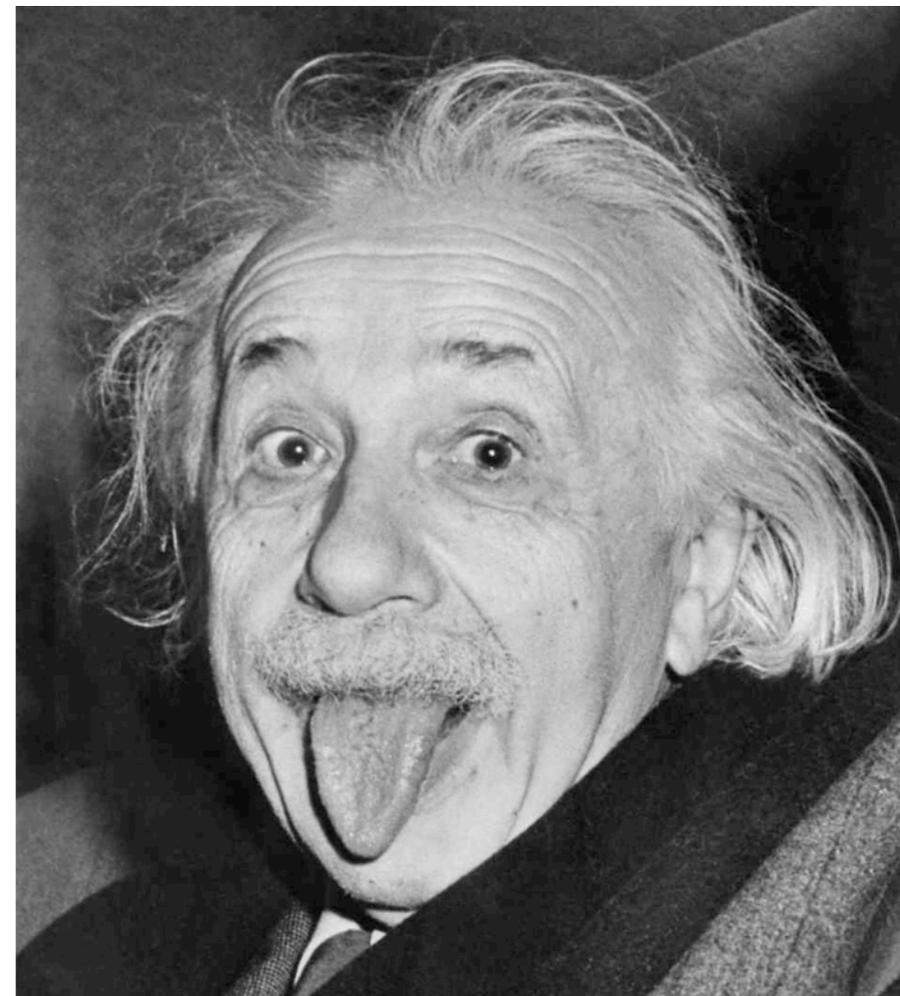
- ➔ The abelian Higgs model couples a Maxwell field with a charged complex scalar subject to a "mexican hat" potential
- ➔ Expanding around the unstable critical point, one sees a massless photon and a "tachyonic" charged scalar
- ➔ Expanding around any of the stable critical points (and thus breaking the symmetry) one finds a massive "photon" and a Higgs boson!

Virtually the same mechanism gives masses to all the (massive) elementary particles in the standard model: intermediate vector bosons, quarks and leptons.

The discovery of a Higgs boson in CERN last summer confirms the Higgs mechanism after almost 50 years since it was first proposed and after more than 20 years of experimental search!

It is remarkable that Nature continues to show mercy at this collective of evolved primates inhabiting a blue planet orbiting a yellow sun and to reveal little by little its innermost secrets to those who strive to discover them.

"Das ewig Unbegreifliche an der Welt ist ihre Begreiflichkeit."



"The most incomprehensible thing about the universe is that it is comprehensible."